6.2 - The Natural Base e

Warmup

1. $\log_7 \sqrt[3]{49}$ $\frac{2}{3}$
2. $\log_3 \sqrt[5]{9}$ $\frac{2}{5}$
3. $\log_{1/2} 8$ -3
4. $\log_{1/3} 27$ -3
5. $\log_2 \sqrt[3]{\frac{1}{4}}$ $-\frac{2}{3}$
6. $\log_{10} \frac{1}{\sqrt{1000}}$ $-\frac{3}{2}$
7. $\log_7 x = 2$ 49
8. $\log_6 x = 3$ 216
9. $\log_9 x = -\frac{1}{2}$ $\frac{1}{3}$
10. $\log_6 x = 2.5$ $36\sqrt{6}$
11. $\log_4 x = -\frac{3}{2}$ $\frac{1}{8}$
12. $\log_{1/9} x = -\frac{1}{2}$ 3

6.2 - The Natural Base e

Euler's Constant

Called the natural base and denoted by *e*.

The actual value is $e = 2.71828182846...$

e can be calculated by the expression $(1+\frac{1}{x})^x$ as x approaches infinity.

x	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1+\frac{1}{x}\right)^x$	2.59374	2.70481	2.71692	2.71815	2.71827	2.71828

6.2 - The Natural Base e

Simplify the expression

1. $e^2 \cdot e^5$

e^7

2. $e^{-3} \cdot e^8$

e^5

3. $\dfrac{12e^5}{36e^2}$

$\dfrac{e^3}{3}$

4. $\dfrac{15e^4}{3e^9}$

$\dfrac{5}{e^5}$

5. $\left(3e^{3x}\right)^2$

$9e^{6x}$

6. $\sqrt{16e^{10x}}$

$4e^{5x}$

6.2 - The Natural Base e

Convert to this form:

$y = a(1 + r)^x$ or $y = a(1 - r)^x$

1. $y = 3e^{2x}$

   $y = 3(e^2)^x$

   $y = 3(7.389)^x$

   $y = 3(1 + 6.389)^x$

   Growth

2. $y = 13e^{-0.123x}$

   $y = 13(e^{-0.123})^x$

   $y = 13(0.884)^x$

   $y = 13(1 - 0.116)^x$

   Decay

6.2 - The Natural Base e

Practice - Convert to this form:

$y = a(1 + r)^x$ or $y = a(1 - r)^x$

1. $y = 2e^{-0.2x}$

   $y = 2(1 - 0.181)^x$

   Decay

2. $y = 5e^{0.6x}$

   $y = 5(1 + 0.822)^x$

   Growth

6.5 - Properties of Logarithms

Logarithm Power Property Proof

$$\log_b x^a = y$$

$$x^a = b^y$$

$$\left(x^a\right)^{1/a} = \left(b^y\right)^{1/a}$$

$$x = b^{y/a}$$

$$\log_b x = \frac{y}{a}$$

$$a \log_b x = y$$

6.5 - Properties of Logarithms

Logarithm Power Property

$$\log_3 x^5 = 5\log_3 x \qquad \log_7 5^4 = 4\log_7 5$$

$$\begin{aligned}\log_7\left((xy)^4\right)^z &= z\log_7(xy)^4 \\ &= 4z\log_7(xy)\end{aligned}$$

Practice

1. $\log_3 m^4$

   $4\log_3 m$

2. $5\log_2 \frac{1}{m^2}$

   $-10\log_2 m$

3. $6\log_4 \frac{m}{\sqrt[3]{m^2}}$

   $2\log_4 m$

6.5 - Properties of Logarithms

Logarithm Product Property Proof

$$m = \log_b x \qquad n = \log_b y$$

$$x = b^m \qquad y = b^n$$

$$x \cdot y = b^m b^n$$

$$x \cdot y = b^{m+n}$$

$$\log_b(xy) = m + n$$

$$\log_b(xy) = \log_b x + \log_b y$$

6.5 - Properties of Logarithms

Logarithm Quotient Property Proof

$$m = \log_b x \qquad n = \log_b y$$

$$x = b^m \qquad y = b^n$$

$$\frac{x}{y} = \frac{b^m}{b^n} = b^{m-n}$$

$$\log_b\left(\frac{x}{y}\right) = m - n$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

6.5 - Properties of Logarithms

Logarithm Product Property

$$\log_3 xy = \log_3 x + \log_3 y$$

$$\log_7 \frac{x}{y} = \log_7(x \cdot y^{-1})$$
$$= \log_7 x + \log_7 y^{-1}$$
$$= \log_7 x - \log_7 y$$

Practice

1. $\log_5 xyz$

   $\log_5 x + \log_5 y + \log_5 z$

2. $\log_2 \frac{xy}{z^3}$

   $\log_2 x + \log_2 y - 3\log_2 z$

3. $\log_4 \left(\frac{x}{y} \cdot (a+b)\right)$

   $\log_4 x - \log_4 y + \log_4(a+b)$

6.5 - Properties of Logarithms

Working with exponents

$$5^{2\log_5 x} = 5^{\log_5 x^2} = x^2$$

Convert to base 2

$$\log_{\frac{\sqrt{2}}{2}} x = \log_{2^{-1/2}} x = \log_2 x^{-2}$$

$$2^{\log_{\frac{\sqrt{2}}{2}} x} = 2^{\log_2 x^{-2}} = x^{-2}$$